

Resistance due to vortex motion in the $\nu = 1$ bilayer quantum Hall superfluid

David A. Huse

Department of Physics, Princeton University, Princeton, NJ 08544

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The longitudinal and Hall resistances have recently been measured for quantum Hall bilayers at total filling $\nu = 1$ in the superfluid state with interlayer pairing, both for currents flowing parallel to one another and for “counterflowing” currents in the two layers. Here I examine the contribution to these resistances from the motion of unpaired vortices in these systems, developing some possible explanations of various qualitative features of these data.

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The $\nu = 1$ interlayer superfluid quantum Hall state occurs in systems where carriers (electrons or holes) are confined to move two-dimensionally in two closely-spaced parallel layers (quantum wells), subject to a perpendicular quantizing magnetic field of near one flux quantum per carrier [1, 2, 3, 4, 5, 6, 7]. It occurs when the tunneling between the layers is negligible, but the layers are close enough together relative to the spacing between carriers that interlayer correlation due to the Coulomb repulsion is strong. The bosons that condense to make a superfluid here are pairs consisting of an electron in one layer and a hole in the other, so they have zero net charge. Recent experiments [5, 6] have looked explicitly at this superfluidity by contacting separately to each layer to produce a current of these interlayer dipoles: counterflowing electrical currents of equal magnitude but opposite direction in the two layers. What is found is that these systems are apparently superfluid only in the zero temperature limit. At nonzero temperature, dissipation is seen, as a nonzero longitudinal resistance, R_{xx} , and this dissipation is found to be a little larger for counterflowing currents as compared to parallel currents that are identically directed in the two layers [5, 6]. As discussed below, this, and also some features of the counterflow Hall resistance, can possibly be understood phenomenologically in terms of the motion of the vortices of this superfluid. Here I will consider only “balanced” bilayers, where the average carrier density is the same in each of the two layers, although the behavior as these bilayers are imbalanced is also interesting [8].

The elementary vortices of this superfluid carry a quantized charge of $\pm e/2$ in addition to their quantized vorticity [3]. Thus there are four types of vortices, with charge and vorticity of either sign. In order to specify these signs unambiguously, we need to set some sign conventions: The carriers are confined to layers parallel to the xy plane, and the perpendicular component of the magnetic field points along the positive z direction. The “top” layer is the layer at larger z . The vector representing a counterflowing current density points in the direction of the electrical current in the top layer and opposite to that in the bottom layer. A positive interlayer electric dipole has positive charge in the top layer and negative charge in the bottom layer. A vortex with positive vorticity has its vorticity (as set by a right-hand

rule on the circulating counterflowing currents) pointing in the positive z direction: viewed from above, this vortex’s electrical currents flow counterclockwise in the top layer and clockwise in the bottom layer. In addition to its charge and vorticity, the elementary vortices also carry an unquantized interlayer electric dipole [3]. With the above sign conventions, the sign of a vortex’s dipole is given by the product of the signs of its charge and vorticity. This is demonstrated, using approximate wavefunctions [3] of these vortices, in the final few paragraphs of the present paper.

In an *ideal* sample with no randomness and exactly at total filling $\nu = 1$, the ground state has no vortices. The vortices in such a two-dimensional superfluid of interlayer dipoles interact logarithmically at large distances, and remain bound in pairs with zero total vorticity up to a nonzero Kosterlitz-Thouless (KT) transition temperature [1, 2, 3]. [Note, the vortex-vortex interaction energies due to the electric charges and dipoles on the vortices fall off with distance, so it is the logarithmic interaction due to the vorticity and the superfluidity that dominates their interaction at large distance.] Below the KT transition temperature a counterflowing current (which is a supercurrent of dipoles) flows without linear-response resistance in such an ideal sample. However, this is not what is seen in the recent experiments [5, 6], where the superfluidity (zero resistance) is observed to be present only in the zero temperature limit. This is likely due to random potential disorder in the samples studied experimentally. A random potential couples to both the electric charge and the dipole moment of the vortices and thus if strong enough can stabilize a ground state with unpaired, pinned vortices in a pattern specific to the particular random potential in each sample. [One can see that the disorder is in some sense rather strong in both of the experimental samples [5, 6] by noting that they enter the insulating phase at magnetic fields just above those that produce the $\nu = 1$ state we are discussing here.] Such a ground state is a type of vortex-glass [7] that is superfluid at zero temperature but not at any positive temperature. This appears to be the situation in the recent experiments [5, 6]. In such a vortex glass state it is the thermally-activated motion of these pinned, unpaired vortices that should dominate the low temperature resistance. Thus it seems worthwhile to look

more closely at the motion of these vortices in response to applied currents.

First, let's look at the *forces* on the vortices due to the currents, at low temperature. Let \mathbf{J} be the current density *per layer*. For *parallel* currents, where \mathbf{J} is the same in both layers, there is a Hall electric field of magnitude $E = 2Jh/e^2$ perpendicular to the current, since we are in a $\nu = 1$ quantum Hall state and the total current density is $2\mathbf{J}$. This Hall electric field couples to the charge $\pm e/2$ of the vortices, producing a force on each vortex that is perpendicular to the current and is of magnitude

$$F_v = Jh/e. \quad (1)$$

Now for *counterflowing* electrical currents, on the other hand, we have a supercurrent density of the interlayer electron-hole pairs (they are bosons) of $\mathbf{J}_d = \mathbf{J}/e$ (net number of pairs per time per length), where $\pm\mathbf{J}$ is the electrical current in the top/bottom layer. As is standard in superfluidity, this supercurrent interacts with the vorticity, producing a Magnus force on the elementary vortices that is perpendicular to the current and of magnitude $F_v = hJ_d = Jh/e$. Thus we find that at this level of approximation the force on a vortex due to a current is of the same magnitude for parallel and counterflowing currents. When the vortices move in a dissipative fashion along the direction of these forces, this produces electrical resistance. The equality of the magnitude of the forces indicates why the longitudinal resistance, R_{xx} , is found to be of (roughly) similar magnitude for both types of currents in the recent experiments [5, 6]. However, although they are of similar magnitude, R_{xx} in fact is measured to be somewhat larger for the counterflowing currents than for the parallel currents in both experiments [5, 6]; I next explore some possible reasons for this difference.

When a vortex moves, there are (at least) two effects in addition to the forces discussed above that might enter. First, the vortex may tend to drift *along* (or opposite to) the current. And, when it moves it is also subject to the Lorentz force due to its charge moving through the magnetic field. Here I will use the term “drift” for the component of a vortex’s motion parallel to the current. I make what appear to be reasonable assumptions about the sign of the direction of this drift. However, better (more microscopic?) arguments to support (or counter) these assumptions would be desirable.

For *parallel currents* the dissipation is reduced if the vortices tend to drift in the same direction as the carriers’ motion. The Lorentz force due to such vortex drift with the carriers opposes the force on the vortex due to the Hall voltage. If the vortex were to drift along at the same speed as the carriers, the net force on it would vanish, just as it does for the carriers. But since the vortices are pinned, the expectation is that their drift speed is less than that of the carriers. The reduction in the force on the vortices due to their drift presumably reduces the rate at which they hop or tunnel in the direction perpendicular to the current, and thus a drift effect of this sign *reduces* the dissipation from what it would be if the

vortices move only perpendicular to the current. It is not clear how one would be able to detect experimentally to what extent the vortices are drifting along the current in this case of parallel currents.

For *counterflowing currents* the carriers are moving in opposite directions in the two layers, so the current by itself cannot dictate the direction in which a vortex drifts. However, this is a current of electric dipoles and each vortex does carry a dipole moment that can determine the direction the vortex drifts. Another way of viewing it is that the density of carriers is imbalanced at the core of each vortex (thus it has an electric dipole), and the direction in which a vortex drifts is the same as the direction of carrier motion in the layer where the vortex has more carriers. Again, there is a Lorentz force due to the motion of this charged vortex, but in this case it adds to the Magnus force due to the supercurrent, increasing the dissipation. For example, let’s consider the case of a vortex with positive charge, dipole and vorticity with the current of dipoles along the positive x direction. In this case the Magnus force is along the negative y direction (its direction is set by the current and the vorticity). When this vortex drifts along the positive x direction (due to its positive dipole), the resulting Lorentz force is also along the negative y direction and adds to the Magnus force. The direction of the Lorentz force is dictated by the current, charge and dipole. For all vortices, the sign of their charge times that of their dipole is equal to the sign of their vorticity, so this addition of the two forces and consequent increase in the dissipation occurs for all four vortex types.

Thus we see that the experimental observation that R_{xx} is larger for counterflowing rather than parallel currents may be due to a tendency of the vortices to drift along with the carriers in a parallel current and/or a tendency of the vortices to drift in the same direction as pairs with the same sign dipole in the case of a counterflowing current.

The motion of the vortices along the current also gives a contribution to the Hall resistance. For a parallel current, the Hall resistance is large, and the small current carried by the charge of the moving vortices is negligible at low temperature compared to the total current. Thus it appears that the contribution of the vortices to the Hall resistance will be too small a correction to detect for a parallel current.

For a *counterflowing current*, on the other hand, the Hall resistance vanishes in the low temperature limit, and vortex motion along the current should give a significant contribution to the Hall resistance at low T . What I find is that the sign of a vortex’s contribution to the counterflow Hall resistance is given by its charge: For example, let’s look again at the vortex with positive charge, dipole and vorticity, with a current of dipoles along the positive x direction. This vortex moves along the negative y direction due to the Magnus and Lorentz forces on it, and it drifts with the current, along the positive x direction. The electric field due to this vortex motion through

the superfluid is perpendicular to the vortex motion and thus has a Hall component along the positive y direction in the top layer. This contribution to the Hall resistance is of the same sign as the conventional Hall resistance of positively charged carriers (holes). The sign of this contribution to the Hall resistance is set by the product of the sign of the vortex's dipole, which determines the direction of the vortex's motion along the current, and the sign of its vorticity, which sets the sign of the resulting electric field. But this product is the sign of the vortex's charge. So, to summarize, the sign of the contribution to the counterflow Hall resistance due the motion of a vortex along the current is the same as that of the conventional Hall resistance for carriers with the same sign charge as the vortex.

When we refer to the net charge of a vortex, this means the difference in charge from a uniform $\nu = 1$ state. Thus for filling $\nu = 1$, there must be an equal number of vortices of positive and negative charge. However, there is in general no particle-hole symmetry, so the core energies, mobilities, and tendencies to drift need not be equal for positively and negatively charge vortices. This allows the sign of the total vortex contribution to the low-temperature counterflow Hall resistance to be set by the specific particle-hole asymmetries of the system. In the experiment on holes [6] the counterflow Hall angle is near zero at low temperature, suggesting that the contributions from the two signs of vortex charge are similar in magnitude and (almost) cancel in this sample. In the data on electrons [5], on the other hand, the counterflow Hall angle at $\nu = 1$ appears to remain nonzero and of the same sign as the conventional Hall effect for electrons, suggesting that the negatively charged vortices dominate in this sample, perhaps due to higher mobility and/or higher tendency to drift along the current.

Next, let's consider the behavior as we move away from total filling $\nu = 1$. If we move well away, the quantum Hall effect is lost, the longitudinal resistance becomes large, the interlayer pairing is lost and the counterflow Hall resistance becomes large and similar in magnitude to the Hall resistance for a parallel current. These same things also happen as the temperature is raised. Thus there is a general trend as one moves away from $\nu = 1$ and $T = 0$ for the counterflow Hall resistance to increase in magnitude to near the conventional value (and sign) for the given density of carriers in each individual layer. At low temperature closer to $\nu = 1$ there is another trend that appears to be in the data [5, 6, 9] and may be due to the vortex motion. For filling larger than but near $\nu = 1$, there are more carriers than flux quanta, and the excess charge will sit on the vortices, so there are now more vortices with the same sign charge as the carriers than there are with the opposite charge. These vortices give a contribution to the counterflow Hall angle of the same sign as the system has well away from $\nu = 1$. For fillings less than $\nu = 1$, the vortices that give the opposite sign contribution are more prevalent. Thus we expect that the counterflow Hall angle will increase towards its

“normal” value as ν is increased from $\nu = 1$, but as ν is decreased the counterflow Hall may first decrease due to the polarization of the vortices' charge before it increases due to the disruption of the pairing. It seems better to use the counterflow Hall *angle* data to look for this effect, since the individual counterflow resistances both have strong and somewhat similar dependences on ν and T . Converting them together into the counterflow Hall angle removes some of this strong dependence. Such an asymmetry of the counterflow Hall angle about $\nu = 1$ is indeed there for the hole samples [9], and appears to be there in Fig. 2b of [5].

Finally, let's look at approximate wavefunctions for the superfluid ground state and its vortices in an ideal, disorder-free bilayer, to determine the signs of the charge, dipole and vorticity of each vortex. Here I follow the paper of Moon, *et al.* [3]. We work in the lowest Landau level (LLL), using Coulomb gauge and the orbitals with angular momentum $m = 0, 1, \dots$ about the origin. Let c_m^\dagger create a carrier in the top layer in the LLL in orbital m , while b_m^\dagger creates a carrier in the same orbital in the bottom layer. Then the ground state of the superfluid is, to first approximation in the inter-carrier Coulomb interaction,

$$|\Psi_0\rangle = \prod_{m \geq 0} \frac{1}{\sqrt{2}} (c_m^\dagger + b_m^\dagger) |0\rangle, \quad (2)$$

where $|0\rangle$ is the “vacuum” of no carriers. In this wavefunction, whenever orbital m is occupied in the top layer, it is empty in the bottom layer, and vice versa. Thus it has interlayer electron-hole pairing. To minimize the interaction energy, the average occupancy of the two layers is equal, and the relative phase between the amplitudes for the carrier being in the two layers is spatially uniform in order to minimize the exchange energy.

To make one type of vortex, instead pair orbital m in the bottom layer with $m + 1$ in the top layer:

$$|\Psi_v\rangle = \prod_{m \geq 0} \frac{1}{\sqrt{2}} (b_m^\dagger + c_{m+1}^\dagger) |0\rangle. \quad (3)$$

This vortex state has on average $1/2$ of a carrier missing in the top layer at the center of the vortex, but the same average density as the ground state in the bottom layer. Since the missing charge is only in the top layer, the net charge and the dipole moment of this vortex have the same sign (in fact, they both have the opposite sign from the charge of the carriers). To zero-th order in the inter-carrier Coulomb interaction this vortex does not have circulating currents, since LLL states in the absence of a potential energy do not carry net current. The superfluid density, and thus the currents, are due to the interactions [3], so to get the sign of the current we must examine the change of this vortex wavefunction due to the interactions, for example in a Hartree-Fock approximation.

Consider the single-particle state in our vortex that is a linear combination of orbital m in the bottom layer and

$m + 1$ in the top layer. Orbital m is concentrated a little closer to the center of the vortex than $m + 1$. This difference in guiding center radius is proportional to $1/\sqrt{m}$, while the radius itself is proportional to \sqrt{m} . This carrier is repelled by all the other occupied states in both layers, so the effective potential it sees has a minimum at a radius somewhere between these two orbitals' centers. This potential (which produces an attraction between the electron and hole) perturbs the two orbitals so that orbital m in the bottom layer is displaced outwards and as a result has a diamagnetic net current, while orbital $m + 1$ in the top layer is displaced inwards and has a paramagnetic net current. Thus we find that for this vortex the currents are paramagnetic in the top layer, and thus, by

the right-hand rule convention I am using, the vorticity is positive, and the product of the signs of the vortex's three attributes, charge, dipole and vorticity, is positive. If we instead pair m in the top layer with $m + 1$ in the bottom layer, this reverses the dipole and the vorticity, but leaves the net charge unchanged, so the product of the three signs remains positive. To change the vortex's net charge, the empty $m = 0$ state is filled with a carrier [3]: this reverses the signs of the charge and the dipole, but leaves the vorticity unchanged.

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